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FUNDAMENTAL EQUATIONS FOR STREAM GAS DYNAMICS

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Аннотация—В статье показано, что основные уравнения струйной термогазодинамики можно вывести из общего баланса для любой физико-механической величины. Основные уравнения струйного движения следуют из законов масс, полных энергий и иервого закона термодинамики. Эти уравнения позволяют также получить одномерное уравнение теплопроводности и магнитной газодинамики.

NOMENCLATURE

l, streamline co-ordinate;
t, time;
$Э$, extensive mechanophysical value;
Δ, increase;
V, volume;
F, cross section;
$э$, $э_v$, specific extensive mechanophysical values;
χ, perimeter of the cross section;
$\dot{y}$, internal specific surface exchange;
$\dot{a}_v$, specific internal release;
ρ, density;
c, flow rate through the cross section;
$\dot{z}$, specific characteristic of non-convective exchange;
$\dot{m}$, mass-flow;
u, specific internal energy;
q, specific heat flow;
p, pressure;
i, specific enthalpy;
T, absolute temperature;
λ, thermal conductivity;
q_{ex}, specific external heat;
H_{ex}, specific external work;
τ_e, shear stress;
s, specific entropy;
v, specific volume;
j, current density;
σ, conductivity;
B, magnetic induction;
μ, magnetic permeability;
h, height;
b, width;
V_{ex}, voltage drop in the external circuit;
ξ, specific kinetic energy;
C_p, specific heat;
K, ratio of specific heats;
R, characteristic constant for a gas.

1. THE GENERAL BALANCE EQUATION

COMPARISON of all the equations governing the processes of stream gas dynamics makes it possible to reduce them to a single general balance equation which determines the distribution along the stream and the change in time of any generalized value.

A number of assumptions are necessary for deriving this equation which, on the one hand, are based on the concept of a streamlined motion and, on the other hand, generalize the laws characterizing the processes accompanying this motion.

The former assumptions are the following: continuity of distribution along the stream and change in time of all the specific and characteristic values, their dependence only upon the curvilinear co-ordinate l of a curvilinear axis of the stream and upon the time t.

The laws of external and internal convective and non-convective exchanges, internal releases and absorptions, etc. relate to the latter group of assumptions. From the continuity condition it should be assumed that in general the corresponding values would be proportional either to volume elements (volume releases, work, etc.) or to time elements. Then the corresponding proportionality coefficient would be in general the known functions of the curvilinear co-ordinate, time, mechanophysical parameters, etc.

While deriving the general balance equation of any value we shall also proceed from such evident equations and relations.

Any value of $\Delta\text{Э}$ corresponding to the stream elements $\Delta V = F\Delta l$ may be determined by the relation at the given moment

$$\Delta\text{Э} = \text{э}_v\Delta V = \text{э}_v F\Delta l \qquad (1.1)$$

where э_v is the density of a given value in the volume ΔV which will be referred to as a specific value as well. In general э_v depends on l and t directly or through any mechanophysical parameters. In (1.1) F represents the useful section of a stream in a considered place at a given moment. In general F is a function of l and t. It should be noted that the curvilinear axis of a stream will be considered immovable, i.e. a stream may pulsate in time but its axis may not change.

Consider the change of $\Delta\text{Э}$ for the time element $\mathrm{d}t$ at l = const. From equation (1.1) we have

$$\mathrm{d}\Delta\text{Э} = \frac{\partial\Delta\text{Э}}{\partial t}\,\mathrm{d}t = \frac{\partial\text{э}_v F}{\partial t}\,\Delta l\,\mathrm{d}t \qquad (1.2)$$

when differentiating with respect to t in particular.

Such a change of $\Delta\text{Э}$ should be obviously the consequence of various physical processes and mechanical motions taking place both in the interior of the volume ΔV and on its front surfaces F and F' and on the lateral surfaces $\Delta F_l = \chi\Delta l$ where χ is a perimeter of the corresponding useful section. Besides, changes of $\Delta\text{Э}$ should be associated with these processes and motions. All the phenomena occurring on the above surfaces and associated with the changes of $\Delta\text{Э}$ may be considered as transfer phenomena through these surfaces for the time $\mathrm{d}t$. Continuing in this way, these phenomena may be expressed by values, proportional to the corresponding surfaces and the element $\mathrm{d}t$:

$$\dot{y}F\,\mathrm{d}t; \quad \dot{y}'F'\,\mathrm{d}t; \quad \dot{y}_l\chi\Delta l\,\mathrm{d}t$$

where $\dot{y}$ and $\dot{y}'$ are internal specific surface exchanges on the surfaces F and F' of the element ΔV and $\dot{y}_l$ is an internal specific surface exchange on its lateral surface. Hence the total surface exchange for $\mathrm{d}t$ will be

$$\dot{y}F\,\mathrm{d}t - \dot{y}'F'\,\mathrm{d}t + \dot{y}_l\chi\Delta l\,\mathrm{d}t = -\left(\frac{\partial\dot{y}F}{\partial l} - \dot{y}_l\chi\right)\Delta l\,\mathrm{d}t. \qquad (1.3)$$

Volume releases in the interior of ΔV will be presented on the same basis by the expression

$$\dot{a}_v\Delta V = \dot{a}_v F\Delta l\,\mathrm{d}t. \qquad (1.4)$$

Considering a continuous stream we have no other conditions which influence the change of $\Delta\text{Э}$ for the time $\mathrm{d}t$. Then from the balance condition of all the mentioned values we shall have a general balance equation:

$$\frac{\partial\text{э}_v F}{\partial t} + \frac{\partial\dot{y}F}{\partial l} = \dot{a}_v F + \dot{y}_l\chi \qquad (1.5)$$

after equating (1.2) with the sum of (1.3) and (1.4) and reduction by $\Delta l\,\mathrm{d}t$. Equation (1.5) relates the change with the time of the distribution density of Э along the stream and the change of internal surface exchange to volume release and external exchange along the co-ordinate in the given section of the stream at a given moment.

Introduction of э, representing the specific distribution of Э over the mass, instead of э_v seems to be more convenient in some applications. It is evident that э_v and э will be related by

$$\text{э}_v = \rho\text{э} \qquad (1.6)$$

where ρ is the density in the volume element ΔV. Then under the same conditions external and internal exchanges are convenient to be expressed as sums consisting of convective and non-convective elements of these exchanges.

Convective elements will be proportional to flow rates through these surfaces and to densities of the value being considered, i.e.

$$\dot{y} = \text{э}_v c + \dot{z}; \quad \dot{y}_l = \eta_l c_l + \dot{z}_l \qquad (1.7)$$

where

c and c_l are flow rates through F and ΔF_l;
э_v and η_l are corresponding densities of Э;
$\dot{z}$ and $\dot{z}_l$ are specific second characteristics of non-convective contributions to exchange.

In view of the said above after substituting (1.6) and (1.7) into equation (1.5) and introducing the mass rate through section F by the formula

$$\dot{m} = \rho c F \qquad (1.8)$$

we obtain

$$\frac{d\vartheta}{dt}+\frac{1}{\rho F}\frac{\partial \dot{z}F}{\partial l}=-\frac{\vartheta}{\rho F}\left(\frac{\partial \rho F}{\partial t}+\frac{\partial \dot{m}}{\partial l}\right)+\frac{\dot{a}_v}{\rho}+\frac{\eta_l c_l+\dot{z}_l}{\rho}\frac{\chi}{F} \quad (1.9)$$

where

$$\frac{d}{dt}=\frac{\partial}{\partial t}+c\frac{\partial}{\partial l}$$

is a substantial derivative sign.

It should be noted that if ϑ_v or ϑ does not possess convective transfer, then expansion of (1.7) as well as (1.6) should not be used and, consequently, balance equation (1.9) is not suitable for such quantities.

2. GENERAL EQUATION OF GAS DYNAMICS OF STREAMLINE MOTION

According to (1.5) and (1.9) we may obtain fundamental equations of stream thermal gas dynamics by corresponding choice of ϑ_v and $\dot{y}$ or ϑ and $\dot{z}$.

(a) Really, considering distribution and change of masses along the stream at $\vartheta_v=\rho$ and assuming in this case absence of non-convective exchanges ($z=\dot{z}_l=0$), we obtain from equations (1.6) and (1.7)

$$\vartheta=1,\quad \dot{y}=\rho c,\quad \dot{y}_l=\rho_l c_l,\quad \dot{a}_v=\dot{\rho}_v \quad (2.1)$$

where ρ_l is the density of external convective exchange and $\dot{\rho}_v$ is the specific mass rate of internal release. Using these assumptions we get from (1.9)

$$\frac{\partial \rho F}{\partial t}+\frac{\partial \dot{m}}{\partial l}=\dot{\rho}_v F+\rho_l c_l \chi \quad (2.2)$$

after multiplication by ρF.

This is a generalized equation of continuity or mass rates for streamline motion (the law of mass conservation).

In case of absence of release and external exchange we transform equation (2.2) into the conventional continuity equation of one-dimensional gas dynamics [1]

$$\frac{\partial \rho F}{\partial t}+\frac{\partial \dot{m}}{\partial l}=0. \quad (2.3)$$

By means of the substantial derivative the latter equation may be also expressed as follows:

$$\frac{d\dot{m}}{dt}=\frac{\dot{m}}{c}\frac{\partial c}{\partial t}=\rho F\frac{\partial c}{\partial t}. \quad (2.3a)$$

(b) Assuming ϑ to be equal to the sum of specific kinetic and specific inner energies of a gas, i.e.

$$\vartheta=\frac{c^2}{2}+u \quad (2.4)$$

and internal non-convective exchange to be equal to the sum of conductive heat transfer and specific power of pressure forces

$$\dot{z}=q+pc. \quad (2.5)$$

Using (2.2) we obtain from (1.9)

$$\frac{di^*}{dt}=\frac{1}{\rho}\frac{\partial p}{\partial t}-\frac{1}{\rho F}\frac{\partial qF}{\partial l}+\frac{\dot{a}_v-i^*\dot{\rho}_v}{\rho}+\frac{\dot{z}_l}{\rho}\cdot\frac{\chi}{F}+\frac{\eta_l-i^*\rho_l c_l}{\rho}\frac{\chi}{F}+\frac{p\partial\ln F}{\rho\partial t} \quad (2.6)$$

where the stagnation enthalpy is equal to

$$i^*=\frac{c^2}{2}+i=\frac{c^2}{2}+u+\frac{p}{\rho}=\vartheta+\frac{p}{\rho}. \quad (2.7)$$

Equation (2.6) is none other than generalization of the equation for stagnation enthalpy (the law of conservation of mechanocaloric energy) known in gas dynamics [1] which in its turn, is obtained from equation (2.6) without external and internal exchange and release at $\dot{\rho}_v=q=\dot{a}_v=0$; $\dot{z}_l=\eta_l=0$, $c_l=0$. Under these conditions we have

$$\frac{di^*}{dt}=\frac{1}{\rho}\frac{\partial p}{\partial t}+\frac{p}{\rho}\frac{\partial\ln F}{\partial t}. \quad (2.8)$$

For stationary motion

$$di^*=0,\quad \text{or } i^*=\frac{c^2}{2}+i=\text{const.} \quad (2.9)$$

This is the law of stagnation enthalpy conservation along the whole length of a stream when the process is stationary without external heat transfer and internal volume release.

In the considered general case from equation (2.6) we may assume

$$\frac{\dot{a}_v}{\rho}=h_u^*;\quad \frac{\eta_l}{\rho}=i_l^*\frac{\rho_l}{\rho}=\left(\frac{c_l^2}{2}+u_l+\frac{p_l}{\rho_l}\right)\frac{\rho_l}{\rho} \quad (2.10)$$

External non-convective exchange may be represented as a sum of thermal and energy exchanges according to

$$\frac{\dot{z}_l}{\rho} \cdot \frac{\chi}{F} = q_{ex} + \dot{H}_{ex}. \tag{2.11}$$

Internal exchange is defined by the Fourier law

$$q = -\lambda \frac{\partial T}{\partial l}. \tag{2.12}$$

Substituting these expressions into (2.6) we obtain the following relation

$$\frac{di^*}{dt} = \frac{1}{\rho}\frac{\partial p}{\partial t} + \frac{1}{\rho F}\frac{\partial}{\partial l}\left(\lambda F \frac{\partial T}{\partial l}\right) + \dot{h}_u^* - i^* \frac{\dot{\rho}_v}{\rho}$$

$$+ (i_l^* - i^*)\frac{\rho_l}{\rho} c_l \frac{\chi}{F} + \dot{q}_{ex} + \dot{H}_{ex} + \frac{p \partial \ln F}{\rho \partial t}. \tag{2.13}$$

In case of stationary motion equation (2.13) is simplified

$$\frac{di^*}{dl} = \frac{1}{\dot{m}}\frac{d}{dl}\left(\lambda F \frac{dT}{dl}\right) + \frac{dh_u^*}{dl} - \frac{i^*}{\rho}\frac{d\rho_v}{dl}$$

$$+ (i_l^* - i^*)\frac{\rho_l c_l}{\rho c}\frac{\chi}{F} + \frac{dq_{ex}}{dl} + \frac{dH_{ex}}{dl} \tag{2.14}$$

at $dl = c\,dt$, where

$$dq_{ex} = \dot{q}_{ex}\,dt = \frac{\dot{q}_{ex}}{c}\,dl; \quad dH_{ex} = \frac{\dot{H}_{ex}}{c}\,dl;$$

$$d\rho_v = \frac{\dot{\rho}_v}{c}\,dl. \tag{2.15}$$

Thus, with stationary streamline motion the whole stagnation enthalpy of a gas will change along the stream because of the following circumstances: internal heat conduction of a gas, heat release in chemical conversions, external non-convective heat and energy transfer and external convective exchange. Thus, for example, for uniflow combustion chambers with continuous secondary air supply and fuel combustion at $dH_{ex} = 0$ (absence of external energy transfer), equation (2.4) presents the total balance of mechanocaloric energy, external exchange and internal release.

(c) Note another interesting particular transformation of the equations obtained in points (a) and (b). Consider (2.8) and (2.3a) together

$$\frac{di^*}{dt} = \frac{1}{\rho}\frac{\partial p}{\partial t} - \frac{p}{\rho F}\frac{\partial F}{\partial t}; \quad \frac{d\dot{m}}{dt} = -\rho F \frac{\partial c}{\partial t}. \tag{2.16}$$

Multiply the first equation by $\dot{m}$, the second one by i^* and sum them up. Then after some simple transformations we get

$$\frac{dJ^*}{dt} = \frac{\partial}{\partial t}\left(\frac{p}{\rho}\dot{m}\right) + э\dot{m}\frac{\partial \ln c}{\partial t} \tag{2.17}$$

where $J^* = \dot{m}i^*$ is the stagnation enthalpy flow.

(d) Now consider the purely thermodynamic quantity i.e. inner energy, assuming $э = u$. In this case, besides specific volume release, $\dot{a}_v$ includes the work of expansion of the initial volume. Dwell upon this work at greater length. It is evident that the change of the volume on the front surfaces F and F' for the time dt may be determined by

$$dV' - dV = F'\,dl' - F\,dl$$

$$= (F'c' - Fc)\,dt = \frac{\partial Fc}{\partial l}\Delta l\,dt.$$

Thus the external work of expansion will be equal to

$$p\frac{\partial Fc}{\partial l}\Delta l\,dt \text{ (with an inverse sign).}$$

Further, volume heat release occurs because of both chemical and other internal conversions and of the work of friction. The latter may be represented by

$$\rho \dot{h}_r \Delta l\,dt = \tau_e c \chi \Delta l\,dt \tag{2.18}$$

where τ_e is the shear stress on the lateral surface of the stream. In this case the specific value of chemical conversion energy would not contain evident kinetic energy of this conversion. Thus, it is equal to

$$h_u = \dot{h}_u^* - \dot{h}_c. \tag{2.19}$$

Then from the above we obtain

$$\dot{a}_v \Delta V\,dt = \rho h_u \Delta V\,dt + \tau_e c \chi \Delta l\,dt - p\frac{\partial Fc}{\partial l}\Delta l\,dt$$

which after reducing by $\Delta V\,\mathrm{d}t$ and rearranging, results in

$$\dot{\alpha}_v = -p\rho\,\frac{\mathrm{d}v}{\mathrm{d}t} + p\,\frac{\partial \ln F}{\partial t} + \rho \dot{h}_u - \frac{p}{\rho}\,\dot{\rho}_v - \frac{p}{\rho}\,\rho_l c_l\,\frac{\chi}{F} + \tau_e c\,\frac{\chi}{F}. \quad (2.20)$$

Other quantities entering equation (2.6) will be equal to $\dot{z} = q$ (conductive heat transfer), $\eta_l = \rho_l c_l$ (specific enthalpy of external convection) and $\dot{z}_l\,(\chi/F) = \dot{q}_{ex}\rho$ (external convective heat transfer). After substituting all these expressions in (1.9), with regard to (2.2) and after reduction by ρ, we find

$$\frac{\mathrm{d}u}{\mathrm{d}t} + p\,\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{p}{\rho}\,\frac{\partial \ln F}{\partial t} - \frac{1}{\rho F}\,\frac{\partial qF}{\partial l} + \dot{h}_u - i\,\frac{\dot{\rho}_v}{\rho} + (i_l - i)\,\frac{\rho_l}{\rho}\cdot\frac{\chi}{F} + \dot{h}_u + \dot{q}_{ex}. \quad (2.21)$$

The obtained equation relates the change of the inner energy and the specific power of gas expansion work to all the forms of heat transported in algebraic sense to the element ΔV for the time $\mathrm{d}t$ with local change of the volume energy in this element (the first law of thermodynamics). When there is no external convection and when $q = -\lambda\,(\partial T/\partial l)$, this equation is simplified according to the Fourier law and takes the form of

$$\frac{\mathrm{d}u}{\mathrm{d}t} + p\,\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{1}{\rho F}\,\frac{\partial}{\partial l}\left(\lambda F\,\frac{\partial T}{\partial l}\right) + \dot{h}_u + \dot{h}_r + q_{ex} + \frac{p}{\rho}\,\frac{\partial \ln F}{\partial t}. \quad (2.22)$$

The latter equation yields the well-known equation of one-dimensional heat conduction

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\lambda}{\rho C_p}\,\frac{\partial^2 T}{\partial l^2} + \dot{h}_u + \dot{h}_r + \dot{q}_{ex} \quad (2.23)$$

when F and $V = \text{const.}$ and $u = cT$.

(e) If, instead of specific inner energy, the enthalpy

$$i = u + \frac{p}{\rho} \quad (2.24)$$

is introduced into equation (2.21), the latter is reduced to

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{1}{\rho}\,\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{p}{\rho}\,\frac{\partial \ln F}{\partial t} - \frac{1}{\rho F}\,\frac{\partial qF}{\partial l} + \dot{h}_u - i\,\frac{\dot{\rho}_v}{\rho} + (i_l - i)\,\frac{\rho_l}{\rho}\,\frac{\chi}{F} + \dot{h}_r + \dot{q}_{ex}. \quad (2.25)$$

Thus we obtain a generalized equation of the first law of thermodynamics for enthalpy in a streamline gas flow. Subtracting (2.25) from (2.13) and assuming the difference of stagnation enthalpy and thermodynamic enthalpy to be equal to the specific kinetic energy, we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{c^2}{2}\right) + \frac{1}{\rho}\,\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{1}{\rho}\,\frac{\partial p}{\partial t} + \dot{h}_c - \frac{c^2}{2}\,\frac{\dot{\rho}_v}{\rho} + \left(\frac{c_l^2}{2} - \frac{c^2}{2}\right)\frac{\rho_l}{\rho}\,c_l\,\frac{\chi}{F} - \dot{h}_r + \dot{H}_{ex}. \quad (2.26)$$

The obtained equation is none other than a generalized balance equation of mechanical energy and to obtain this equation as well as equation (2.25), new determinations of quantities, entering the basic balance equation, are not necessary. Thus, the two equations follow from (2.2), (2.13) and (2.21). In other words, the law of mechanical energy balance is a direct result of the laws for balance of masses, mechano-caloric energy and inner energy, i.e. it follows from the mass and total energy conservation law and the first law of thermodynamics for gases.

It is easy to see that when external convective exchange and inner release are absent, the known generalized Lagrange–Bernoulli equation arises from equation (2.26) which after introduction

$$\mathrm{d}h_r = \dot{h}_r\,\mathrm{d}t = \frac{\dot{h}_r}{c}\,\mathrm{d}l; \quad \mathrm{d}H_{ex} = \frac{\dot{H}_{ex}}{c}\,\mathrm{d}l \quad (2.27)$$

takes the form of

$$\mathrm{d}\left(\frac{c^2}{2}\right) + \frac{\mathrm{d}p}{\rho} + \mathrm{d}h_r = \frac{1}{\rho}\,\frac{\partial p}{\partial t}\,\mathrm{d}t + \mathrm{d}H_{ex}. \quad (2.28)$$

This equality may be integrated and reduced to

$$H_{ex} = \frac{c^2 - c_0^2}{2} + \int_{p_0}^{p}\frac{\mathrm{d}p}{\rho} + h_r \quad (2.29)$$

provided that motion is stationary.

(f) As the third result of the obtained equations, deduce the equation for specific entropy determined by the equalities

$$T\,\mathrm{d}s = \mathrm{d}u + p\,\mathrm{d}v = \mathrm{d}i - v\,\mathrm{d}p. \quad (2.30)$$

Then from (2.21) and (2.25) it follows that

$$T\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{p}{\rho}\frac{\partial \ln F}{\partial t} - \frac{1}{\rho F}\frac{\partial qF}{\partial l} + \dot{h}_u - i\frac{\dot{\rho}_v}{\rho} + (i_l - i)\frac{\rho_l}{\rho}\frac{\chi}{F} + \dot{h}_u + \dot{q}_{ex}. \quad (2.31)$$

Equation (2.31) shows that its right side may be transformed into zero by special choice of values. We are naturally interested not in this specially chosen case $\mathrm{d}s = 0$, but in general conditions when $\mathrm{d}s = 0$. Therefore we exclude all the terms of external convective exchange and inner release from (2.31). Then, assume q to be determined by the Fourier law. Under these conditions equation (2.31) may be represented as:

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{\lambda}{\rho}\left(\frac{1}{T}\frac{\partial T}{\partial l}\right)^2 + \frac{\dot{h}_r}{T} + \frac{1}{\rho}\frac{\partial}{\partial l}\left(\frac{\lambda}{T}\frac{\partial T}{\partial l}\right) + \frac{\lambda}{\rho T}\frac{\partial T}{\partial l}\frac{\partial \ln F}{\partial l} + \frac{p}{\rho T}\frac{\partial \ln F}{\partial t} + \frac{\dot{q}_{ex}}{T}. \quad (2.32)$$

The latter equation shows that when external and internal conductive heat exchanges are absent ($\lambda = 0$, $\dot{q}_{ex} = 0$) and when motion is stationary $(\mathrm{d}s/\mathrm{d}t) = (\dot{h}_r/T) > 0$ or when a stream is thermally insulated and there is no heat conduction, specific entropy of any gas element of a stationary stream increases with time, and if internal friction forces are also absent, entropy remains constant (the second law of thermodynamics). If these conditions are not observed, internal friction (dissipative function of friction) and internal heat conduction (dissipative function of heat conduction) always produce two terms in the right side of equation (2.32).

3. SOME SPECIFIC EQUATIONS OF STREAM GAS DYNAMICS FOLLOWING FROM GENERAL EQUATION

Consider a stationary linear flow of a neutral conducting gas in a direct magnetic field normal to the flow. In this case the value of internal release [in the fundamental equation (2.6)] would include Joule heat and Lorenz force work. Thus, for the stream element assumed to be flat, with cross section $F = bh$ where b is the constant width of the stream and h is the height, dependent on the co-ordinate $l = x$ (the rectlinear axis of the stream), we find

$$\dot{a}_v \Delta V\,\mathrm{d}t = \frac{j^2}{\sigma} hb\Delta l\,\mathrm{d}t - jBcbh\Delta l\,\mathrm{d}t + \rho\dot{h}_u^* F\Delta l\,\mathrm{d}t$$

for internal release.

Hence

$$\dot{a}_v = \frac{j^2}{\sigma} - jBc + \rho\dot{h}_u^* \quad (3.1)$$

where j is the current density, σ is the conductivity, B is the magnetic induction equal to the product of magnetic permeability μ and the magnetic field strength H. Substituting (3.1) into (2.6) we get

$$\frac{\mathrm{d}i^*}{\mathrm{d}t} = -\frac{1}{\rho h}\frac{\partial qh}{\partial x} + \frac{j^2}{\rho\sigma} - \frac{\mu jHc}{\rho} + \dot{h}_u^* - i^*\frac{\dot{\rho}_v}{\rho} + \frac{\dot{z}_l}{\rho}\cdot\frac{2}{h} + (i_l^* - i^*)\frac{\rho_l}{\rho}c_l\frac{2}{h} \quad (3.2)$$

or when $\mathrm{d}t = (\mathrm{d}x/c)$ and $q = -\lambda\,(\partial T/\partial x)$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{c^2}{2} + i\right) = \frac{\lambda}{\rho c h}\cdot\frac{\partial}{\partial x}\left(\lambda h\frac{\partial T}{\partial x}\right) + \frac{j}{\rho c}\left(\frac{j}{\sigma} - \mu Hc\right) + \frac{1}{c}\left(\dot{h}_u^* - i^*\frac{\dot{\rho}_v}{\rho}\right) + (i_l^* - i^*)\frac{2\rho_l c_l}{\rho c h} + \frac{\mathrm{d}q_{ex}}{\mathrm{d}x} + \frac{\mathrm{d}H_{ex}}{\mathrm{d}x}. \quad (3.3)$$

Continuity equation (2.2) and the equation of mechanical energy balance (2.26) should be added to this equation. Then it takes the form of

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{c^2}{2}\right) + \frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}x} = -\frac{\mathrm{d}h_r}{\mathrm{d}x} - \frac{\mu jH}{\rho} + \left(\dot{h}_c - \frac{c^2}{2}\frac{\dot{\rho}_v}{\rho}\right)\frac{1}{c} + \frac{c_l^2 - c^2}{2}\frac{\rho_l}{\rho}\frac{c_l}{c}\frac{\chi}{F} + \frac{\mathrm{d}H_{ex}}{\mathrm{d}x} \quad (3.4)$$

when Lorenz forces are taken into account. A relation of the current density, velocity and magnetic induction, that will be determined by the problem conditions, should be added to the above equations. For instance [2]

$$j = \frac{\sigma}{h}(cBh - V_{ex}) \tag{3.5}$$

is assumed for a magnetic nozzle where V_{ex} is the constant voltage drop in the external circuit.

In the particular case when all the exchanges and releases, except electrical magnetic ones, are absent and when friction forces are neglected, the above equations for ideal gas are reduced to such a set of equations at $\xi = (c^2/2)$

$$\frac{di}{dx} = Kj\left(\frac{B}{\rho} - \frac{V_{ex}}{\dot{m}}\right);$$

$$\frac{d\xi}{dt} = Kj\left(\frac{K-1}{K}\frac{V_{ex}}{\dot{m}} - \frac{B}{\rho}\right)$$

$$j = \sigma B\left[\sqrt{(2\xi)} - \frac{V_{ex}}{h\sqrt{(2\xi)}}\right],$$

$$\dot{m} = \rho h\sqrt{(2\xi)} = \text{const.}, \; \xi = \frac{c^2}{2}, \; p = \frac{K-1}{K}\, i\rho. \tag{3.6}$$

This set relates seven quantities p, ρ, ξ, i, j and B which depend on x in the general case. Thus, for complete certainty two equations more should be added or two values should be restricted by certain conditions of their change along the x-axis.

From these equations an expression for entropy change is easily obtained

$$\frac{ds}{dt} = \frac{j}{T}\left(\frac{B}{\rho} - \frac{V_{ex}}{\dot{m}}\right) \tag{3.7}$$

and for pressure change

$$\frac{dp}{dx} = (K-1)j\left(\frac{B}{\rho} - \frac{V_{ex}}{\dot{m}}\right). \tag{3.8}$$

From (3.7) and (3.6) we find that

$$\frac{ds}{di} = \frac{1}{KT} = \frac{C_p}{Ki} = \frac{R}{(K-1)i}$$

which after integrating yields

$$s = s_0 + \frac{R}{K-1}\ln\frac{i}{i_0} \tag{3.9}$$

On the other hand, entropy for an ideal gas is given by the following equation, [1]:

$$s = s_0 + \frac{R}{K-1}\left(\ln\frac{T}{T_0}\right) + \ln\left(\frac{\rho}{\rho_0}\right)^{-(K-1)}. \tag{3.10}$$

Comparison of (3.9) and (3.10) obtains such a relation between i and ρ

$$\frac{i}{i_0} = \frac{i}{i_0}\left(\frac{\rho}{\rho_0}\right)^{-(K-1)}$$

which is possible only at $\rho = \rho_0 = \text{const.}$, i.e. when the process is isobaric.

All the above shows that fundamental equations and consequences of stream gas dynamics would be derivable from the general balance equation for any mechanophysical value. If the present approach is assumed, fundamental equations follow from mass laws, total energies and the first law of thermodynamics or, in other words, the general case of one dimensional stream seems to be possible to expound without using the conventional equation of hydrodynamics and Newton laws. These equations allow one to obtain the one-dimensional equation of heat conduction and magnetic gas dynamics.

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Abstract—It is shown in the paper that fundamental equations of stream gas dynamics may be derived from the general balance equation for any mechanophysical value. Fundamental equations follow from mass laws, total energies and the first law of thermodynamics. These equations make it possible to obtain a one-dimensional equation of heat conduction and magnetic gas dynamics.

Résumé—On montre dans cet article que les équation fondamentales de la dynamique des écoulements gazeux peuvent être déduites de l'équation générale d'équilibre pour toute valeur mécanóphysique. Les équations fondamentales sont établies à partir des lois de conservation de la masse, de l'énergie et du premier principe de thermodynamique. Ces équations permettent d'obtenir une équation unidimensionnelle pour la conduction thermique et la magnétohydrodynamique.

Zusammenfassung—In der Arbeit wird dargelegt, dass aus der allgemeinen Gleichgewichtsbeziehung für beliebige mechanisch-physikalische Werte grundsätzliche Gleichungen der Gasdynamik abgeleitet werden können. Grundgleichungen ergeben sich aus den Beziehungen für Masse und Gesamtenergie sowie dem ersten Hauptsatz der Thermodynamik. Diese Beziehungen führen auf eine eindimensionale Gleichung für Wärmeleitung und Magnetogasdynamik.